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MAY 77 A FEUER, B R BARMISH, A S MORSE AFOSR-77-3176

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AN UNSTABLE DYNAMICAL SYSTEM ASSOCIATED
WITH MODEL REFERENCE ADAPTIVE CONTROL*

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ABSTRACT

It is shown that a certain system of differential equations of importance to the proof of stability of the adaptive system proposed in [1], admit unbounded solutions. The implication of this result is that a much more elaborate argument than heretofore thought necessary is required to prove that the adaptive system of [1] is stable, if it is stable at all.

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In studying the asymptotic behaviour of the adaptive control system proposed in [1], one encounters equations of the form

$$\dot{\eta} = -\eta + \phi(t)\delta \quad (1)$$

$$\dot{\delta} = -\phi(t)\eta \quad (2)$$

$$\dot{w}_1 = -w_1 + \phi^2(t)\eta \quad (3)$$

where as in [1], η, δ, w_1 and ϕ are the augmented error, parameter error, auxiliary signal and sensitivity function respectively, of the adaptive system. These particular equations result if one assumes (for simplicity) that $D_m(p) = (p+1)D_w(p)$, $D_f(p) = p+1$ and $N=1$, where D_m, D_w, D_f and N are as defined in [1].

To prove that the adaptive system of [1] is stable, it is necessary to show that η, δ and w are bounded. Since the structure of the adaptive system makes it difficult to deduce very much about ϕ unless η, δ and w_1 are known a priori to be bounded, the approach in [1] and elsewhere has been to try to establish the boundedness of η, δ, w_1 without first assuming that ϕ is bounded. To get some idea of what's involved, observe that the time function

$$\alpha = \frac{1}{2} (\eta^2 + \delta^2) \quad (4)$$

satisfies

$$\dot{\alpha} = -\eta^2 \quad (5)$$

from which boundedness of η and δ directly follow. This and (2) imply that the output of any stable first-order linear system with input $\phi\eta$, is bounded. It is thus reasonable to expect that w_1 , the output of a stable first-order linear system forced by $\eta\phi^2$, will be bounded as well. The following counterexample shows that this is not the case.

Proposition: If

$$\dot{\phi} = \dot{\theta} + (\sin \theta)(\cos \theta) \quad (6)$$

where

$$\theta = e^{-t} \sin^2(e^{6t}) \quad (7)$$

then there exists an unbounded solution to (1)-(3).

Since the sensitivity function ϕ actually generated by the adaptive system of [1] is not completely arbitrary, the preceding is not quite a counterexample to the claim of stability of the adaptive system proposed in [1]. On the other hand, the example does imply that a much more elaborate argument involving the differential equations which generate ϕ is required to prove that the adaptive system is stable, if it is stable at all.

To prove the proposition, first observe from (1), (2) and (7), with $\eta(0) = \sin(\sin 1)$ and $\delta(0) = \cos(\sin 1)$, that $\eta = \zeta \sin \theta$ where $\delta = \zeta \cos \theta$ and

$$\zeta(t) = e^{-\int_0^t \sin^2 \theta(\tau) d\tau} \quad (8)$$

Hence

$$\eta \dot{\phi}^2 = \zeta \dot{\phi}^2 \sin \theta \quad (9)$$

The definition of θ in (5) implies that

$$\sin \theta \geq \theta^2/2 \geq 0 \quad (10)$$

and that $\theta^2 \leq e^{-2t}$; from the last inequality, the trigonometric relation $\sin^2 \theta \leq \theta^2$ and (8) it follows that $\zeta(t) \geq c_1 \equiv e^{-1/2}$. This (9) and (10) thus yield $\eta \dot{\phi}^2 \geq c_1 \dot{\phi}^2 \sin \theta$. Using (4) to substitute for $\dot{\phi}$, we obtain

$$\eta \dot{\phi}^2 \geq c_1 \sin \theta (\dot{\theta}^2 + 2\dot{\theta}(\sin \theta)(\cos \theta) + (\sin^2 \theta)(\cos^2 \theta)) \quad (11)$$

Now observe that from (5), $\dot{\theta} + \theta = 6e^{5t} \sin(2\gamma)$ where

$$\gamma = e^{6t} \quad (12)$$

Hence

$$\dot{\theta}^2 \sin \theta = (6e^{5t} \sin 2\gamma)^2 \sin \theta - (2\theta\dot{\theta} + \theta^2)(\sin \theta) \quad (13)$$

If we now define

$$\left. \begin{aligned} b_1 &= ((\sin^2 \theta)(\cos^2 \theta) - \theta^2) \sin \theta c_1 \\ b_2 &= (2/3 \sin^3 \theta + 2(\theta \cos \theta - \sin \theta)) c_1 \end{aligned} \right\} \quad (14)$$

then using (11) and (12),

$$\eta\dot{\phi}^2 \geq c_1 \sin \theta (6e^{5t} \sin 2\gamma)^2 + b_1 + \dot{b}_2 \quad (15)$$

From (10), and then (7) and (12)

$$\begin{aligned} \sin \theta (6e^{5t} \sin 2\gamma)^2 &\geq 18 \theta^2 e^{10t} \sin^2 2\gamma \\ &= 18e^{8t} (\sin^4 \gamma) (\sin^2 2\gamma) \\ &= 18e^{8t} (1 - \cos^2 \gamma)^2 (\sin^2 2\gamma) \\ &\geq 18e^{8t} (1 - 2 \cos^2 \gamma) (\sin^2 2\gamma) \\ &= -18e^{8t} (\cos 2\gamma) (\sin^2 2\gamma) \\ &= -\frac{1}{2} e^{2t} \frac{d}{dt} (\sin^3 2\gamma) \end{aligned}$$

This and (15) thus show that

$$\eta\dot{\phi}^2 \geq -c_2 e^{2t} \frac{d}{dt} (\sin^3 2\gamma) + b_1 + \dot{b}_2 \quad (16)$$

where $c_2 = c_1/2 > 0$.

From the easily verified identities

$$-\int_0^t e^{3\tau} \frac{d}{d\tau} (\sin^3 2\gamma) d\tau = -e^{3t} \sin^3 2\gamma + 3 \int_0^t e^{3\tau} \sin^3 2\gamma d\tau$$

and

$$3e^{3t} \sin^3 2\gamma = 3/4 b_3 + 1/4 \dot{b}_3$$

where

$$b_3 \equiv e^{-3t} (1/3 \cos^3 2\gamma - \cos 2\gamma) \quad (17)$$

it follows that

$$\int_0^t e^{3\tau} \frac{d}{d\tau} (\sin^3 2\gamma) d\tau = -e^{3t} \sin^3 2\gamma + \frac{1}{4} \int_0^t (3b_3 + \dot{b}_3) d\tau$$

Thus from (16)

$$\int_0^t e^{(\tau-t)} \eta(\tau) \phi^2 d\tau \geq -c_2 e^{2t} \sin^3 2\gamma + b(t) \quad (18)$$

where

$$b(t) = -\frac{e}{4} c_2 \int_0^t (3b_3 + \dot{b}_3) d\tau + \int_0^t e^{(\tau-t)} (b_1 + \dot{b}_2) d\tau$$

Since (14) and (17) show that b_1, b_2 and b_3 are bounded, $b(t)$ is a bounded function as well.

Thus if we take $w_1(t)$ to be the zero initial condition solution to (3), then from (18), and (12)

$$w_1(t) \geq -c_2 e^{2t} \sin^3 (2e^{6t}) + b(t).$$

Clearly $w_1(t)$ is unbounded. \square

REFERENCE

- [1] R. V. Monopoli, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. Auto. Control, AC-19 (5), October, 1974, pp. 474-484.

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